Teaching guide - Boolean algebra

This workbook is designed to help you understand how to simplify Boolean algebra expressions. Written for use with the AQA A-level Computer Science specification.

Contents

Introduction to Boolean algebra

1. True is represented by the number 1.
2. False is represented by the number 0.
3. A variable is normally represented, for convenience by a single letter.
4. The OR logic gate is represented by a '+' symbol. Example: $A + B$ means A OR B.
5. The AND logic gate is represented by a '.' symbol. Example: $A.B$ means A AND B.
6. $A.\overline{B} + \overline{A}.B$ is an equation for A XOR B. Alternatively the XOR logic gate can be represented by a $\oplus$ symbol. Example: $A \oplus B$ means A XOR B.
7. The NOT logic gate is represented using an overbar. Example: $\overline{A}$ means NOT A.
8. There are no special symbols used to represent NAND and NOR. These logic gates are represented using combinations of the other logic gates.
9. The expression $\overline{A + B}$ represents A NOR B.
10. The expression $\overline{A.B}$ represents A NAND B.

OR

The OR logic gate takes two inputs returning a value of true (1) if any of the inputs are true (1).

$0 + 0 = 0$ $\quad$ $0 + 1 = 1$

$1 + 0 = 1$ $\quad$ $1 + 1 = 1$

AND

The AND logic gate takes two inputs returning a value of true (1) if both the inputs are true (1).

$0.0 = 0$ $\quad$ $0.1 = 0$

$1.0 = 0$ $\quad$ $1.1 = 1$

NOT

The NOT logic gate takes one input and returns the opposite of that input.

$\overline{0} = 1$

$\overline{1} = 0$

XOR

The XOR logic gate takes two inputs returning a value of true (1) if exactly one input is true (1).

$0 \oplus 0 = 0$ $\quad$ $0 \oplus 1 = 1$

$1 \oplus 0 = 1$ $\quad$ $1 \oplus 1 = 0$

Order of precedence

In algebraic expressions there is an order of precedence for the operations. BIDMAS is a mnemonic used to help remember this order – Brackets, Indices, Division/Multiplication, Addition/Subtraction.

There is also an order of precedence for the operations used in Boolean algebra. The order of precedence is shown below (highest priority first):

1. Brackets
2. NOT
3. XOR
4. AND
5. OR

The important one to remember is that AND has higher priority than OR. So the expression $A.B + C$ is equivalent to the truth table:

A	B	C	A.B	A.B+C
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

That is $A.B + C$ means the same as $(A.B) + C$ but the brackets are omitted as they are not necessary as AND is a higher precedence operator than OR.

If the OR operator is supposed to be applied first then brackets are needed around this part of the expression. The equivalent truth table is:

A	B	C	B+C	A.(B+C)
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

You can see from these truth tables that the order that operators are applied in has a significant impact on the final result. If you want an OR to take priority over an AND then brackets are needed.

Commutative laws

These laws state that the order of the operands does not matter (with some operators). You will be very familiar with these laws from algebraic expressions in Maths – they are so obvious that you probably don't think about them at all e.g. 3 x 5 is the same as 5 x 3.

The commutative laws in Boolean algebra are:

$A.B = B.A$

$A + B = B + A$

$A \oplus B = B \oplus A$

Associative laws

These laws state that when all the operators are the same the order they are applied in does not matter (for some operators). These laws are also used in algebraic expressions and are so obvious that you probably don't think about them at all e.g. (3 x 5) x 4 = 3 x (5 x 4).

The associative laws in Boolean algebra are:

$A.(B.C) = (A.B).C$

$A + (B + C) = (A + B) + C$

$A \oplus (B \oplus C) = (A \oplus B) \oplus C$

Simplifying Boolean expressions part 1

Simplifying an expression means rewriting the expression in a way that uses fewer logic gates but keeping exactly the same functionality.

Basic rule 1: apply the OR operator to an expression and the value FALSE

$A + 0 = A$

Proof using a truth table

A	A	A+0
0	0	0
1	0	1

You can see that every row in the column A + 0 is always the same as the equivalent row in the column A so they are equivalent expressions.

This means that anytime we OR an expression with False (0) that can always be simplified to just the expression.

Basic rule 2: apply the OR operator to an expression and the value TRUE

$A + 1 = 1$

Proof using a truth table

A	1	A+1
0	1	1
1	1	1

You can see that every row in the column $A + 1$ contains a 1, so when you OR an expression with True (1) that can always be simplified to just True (1).

Basic rule 3: apply the OR operator to an expression and another copy of the same expression

$A + A = A$

Proof using a truth table

A	A	A+A
0	0	0
1	1	1

You can see that every row in the column $A + A$ is always the same as the equivalent row in the column A so they are equivalent expressions. Whenever an OR operator is given the same expression twice that can be simplified to just that expression.

Basic rule 4: apply the OR operator to an expression and the inverse of that expression

$A + \overline{A} = 1$

Proof using a truth table

A	$\overline{A}$	$A + \overline{A}$
0	1	1
1	0	1

You can see that every row in the column $A + \overline{A}$ is always True (1), so when you OR an expression with its inverse that can always be simplified to just True (1).

Basic rule 5: apply the AND operator to an expression and the value FALSE

$A.0 = 0$

Proof using a truth table

A	0	A.0
0	0	0
1	0	0

You can see that every row in the column $A.0$ is always 0 (False), so when you AND an expression with False (0) that can always be simplified to just False (0).

Basic rule 6: apply the AND operator to an expression and the value TRUE

$A.1 = A$

Proof using a truth table

A	1	A.1
0	1	0
1	1	1

You can see that every row in the column $A.1$ is always the same as the equivalent row in the column A so they are equivalent expressions.

This means that anytime we AND an expression with True (1) that can always be simplified to just the expression.

Basic rule 7: apply the AND operator to an expression and another copy of the same expression

$A.A = A$

Proof using a truth table

A	A	A.A
0	0	0
1	1	1

You can see that every row in the column $A.A$ is always the same as the equivalent row in the column A so they are equivalent expressions. Whenever an AND operator is given the same expression twice that can be simplified to just that expression.

Basic rule 8: apply the AND operator to an expression and the inverse of that expression

$A.\overline{A} = 0$

Proof using a truth table

A	$\overline{A}$	$A.\overline{A}$
0	1	0
1	0	0

You can see that every row in the column $A.\overline{A}$ is always False (0), so when you AND an expression with its inverse that can always be simplified to just False (0).

Basic rule 9: apply the NOT operator to an expression twice

$\overline{\overline{A}} = A$

Proof using a truth table

A	$\overline{A}$	$\overline{\overline{A}}$
0	1	0
1	0	1

You can see that every row in the column $\overline{\overline{A}}$ is always the same as the equivalent row in the column A so they are equivalent expressions. Whenever a NOT operator is applied twice to an expression that can be simplified to just that expression.

Taking care with cancelling NOTs

Two NOTs only cancel each other out if they are over exactly the same part of an expression.

$\overline{\overline{D}} = D$ is correct.

$\overline{\overline{A + B}} = A + B$ is correct.

But $\overline{A + \overline{B.C}} = A + B.C$ is **not** correct as the NOTs are not over the same part of the expression.

$\overline{\overline{A} + \overline{B.C}} = A + B.C$ is **not** correct as the NOTs are not over the same part of the expression.

$\overline{\overline{A + B.C}} = A + B.C$ is correct.

$A + \overline{\overline{B.C}} = A + B.C$ is correct as the NOTs are over exactly the same part of the expression.

Exercise 1

Using just the basic rules 1-9, simplify the expressions below **showing your working out** when there is more than one stage to the simplification. Sometimes the commutative and associative laws may also help you answer the questions.

1) $A + A$

2) $B + B$

3) $A.B + A.B$

4) $D.F + D.F + G$

5) $D.E + A.\overline{B} + D.E$

6) $A.A$

7) $H.H$

8) $(A + B).(A + B)$

9) $X.Y.X$

10) $B.1$

11) $C + 0$

12) $A.B.A.C.1$

13) $D + 0$

14) $A.B + 0$

15) $E.1$

16) $E.0$

17) $(A + A.B).0$

18) $A.B.1$

19) $\overline{\overline{D}}$

20) $\overline{\overline{B + B}}$

21) $\overline{C + \overline{\overline{D}}}$

22) $A + \overline{B.\overline{B}}$

23) $\overline{A + \overline{B}}$

24) $A + A + 1$

25) $A.B.1.0$

26) $A + \overline{\overline{\overline{B}}}$

For the questions below, complete truth tables to prove that the statements are true – do this without looking at the previous pages. When finished check your answers using the truth tables on the previous pages.

27) $E.0 = 0$

28) $E.1 = E$

29) $E.E = E$

30) $E.\overline{E} = 0$

31) $E + E = E$

32) $\overline{\overline{E}} = E$

33) $E + 1 = 1$

34) $E + 0 = E$

35) $E + \overline{E} = 1$

Simplifying Boolean expressions part 2

Basic rule 10: A + A.B = A

Proof using a truth table

A	B	A.B	A+A.B
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

You can see that every row in the column $A + A.B$ is always the same as the equivalent row in the column A so they are equivalent expressions.

Basic rule 11: A. (A + B) = A

Proof using a truth table

A	B	A+B	A.(A+B)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

You can see that every row in the column $A.(A + B)$ is always the same as the equivalent row in the column A so they are equivalent expressions.

Alternative proof for basic rule 11

$A.(A + B)$

$= A.A + A.B$ Expand the brackets (see page 14 for more details)

$= A + A.B$ Apply basic rule 7

$= A$ Apply basic rule 10

Exercise 2

Using just the basic rules 1-11, simplify the expressions below **showing your working out.** Sometimes the commutative and associative laws may also help you answer the questions.

1) $C + C.D$

2) $D + C.D.B$

3) $A.(C + A)$

4) $D.F + D.1$

5) $E.F.(E.F + D)$

6) $A.A + A.1 + B.\overline{B}$

7) $A.B.B.C + A.0.B + A.C.B.D$

8) $A.B.\left(0 + \overline{A}\right) + 1$

9) $\overline{D}.(\overline{E} + \overline{D})$

10) $F.(F + 1)$

11) $E.F + F.G.E$

12) $\overline{\overline{B.\overline{C}}} + \overline{C}$

13) $A.B.(A + A.B)$

14) $A.B.\overline{C} + A.\overline{C}$

15) $A.\overline{D}.(C + A.\overline{D})$

16) $\overline{C.\overline{D}.\overline{D}.E} + \overline{C.E}$

17) $A + B.A + B.A.C$

18) $E.(E + 1)$

19) $\overline{D} + D.0$

20) $A + A.\overline{A}$

21) $\overline{\overline{\overline{B} + \overline{B}.\overline{B}}}$

22) $C.A.B.(A.B + D)$

23) $\left(A.\overline{G} + A.1\right).A$

24) $B.A + A$

25) $A.(A + B).\overline{1}$

26) $A.B.(A.B + E) + \overline{\overline{\overline{A.\overline{\overline{B}}}}}$

For the questions below, write out truth tables to prove that the statements are true – do this without looking at the previous pages in the workbook. When finished check your answers using the truth tables on the previous pages.

27) $B.\overline{B} = 0$

28) $B.1 = B$

29) $\overline{\overline{B}} = B$

30) $B + \overline{B} = 1$

31) $B + B = B$

32) $B + 1 = 1$

33) $B.B = B$

34) $B + 0 = B$

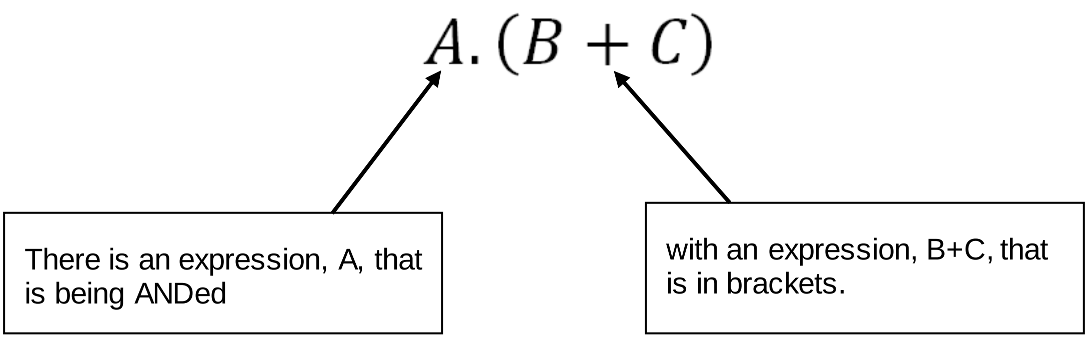
35) $B.0 = 0$

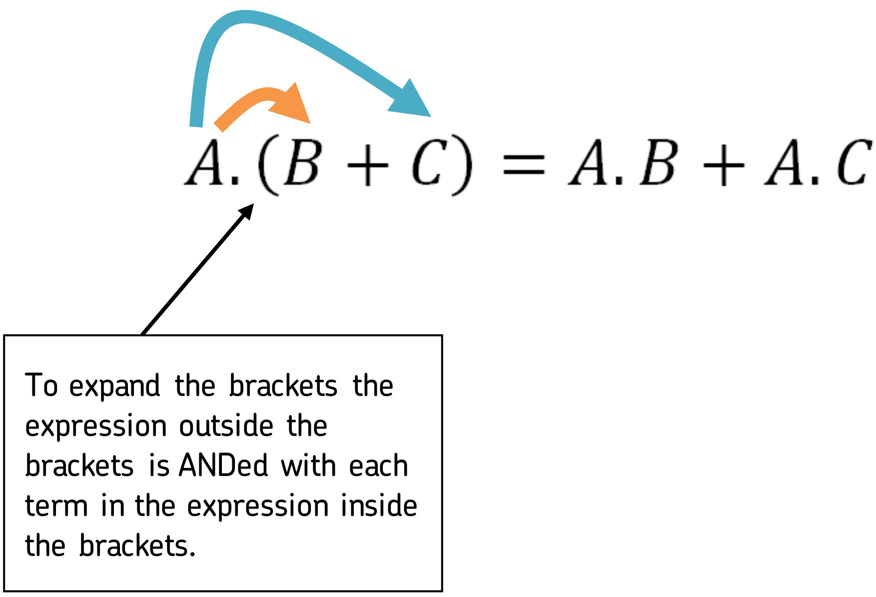
Simplifying Boolean expressions part 3: expanding brackets and factorising

Expanding brackets

In algebraic expressions you will be familiar with expanding the brackets ("multiplying out") e.g. 3(y+7) = 3y + 21. It is also possible to expand brackets in Boolean algebra expressions when an expression is ANDed with an expression enclosed in brackets. This can often help to simplify an expression (though sometimes it might not – just because you can expand brackets does not mean it is always right to do so).

Worked example

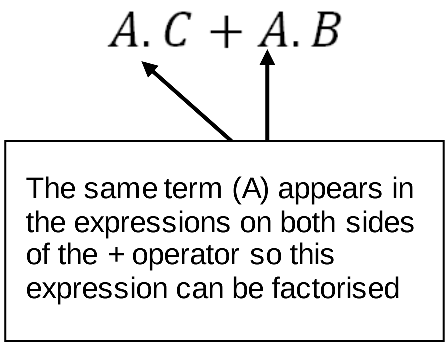


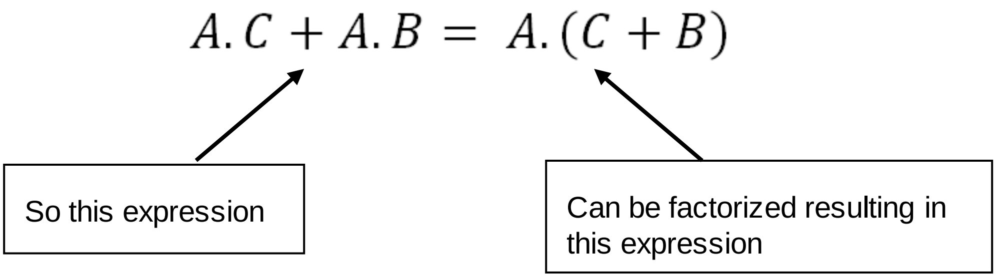


Factorising

In algebraic expressions you will have seen that sometimes an expression can be factorised – brackets are added into an expression and a common factor to all the terms inside the brackets is put outside the brackets e.g. 3y+6 = 3(y+2). Factorising is the reverse of expanding the brackets. It is also possible to factorise many Boolean algebra expressions. Sometimes doing so can help to simplify an expression.

Worked example





Exercise 3

For questions 1-6, expand the brackets in the expressions.

Do **not** simplify the resulting expressions.

1) $C.(D+B)$

2) $C.D.(B+A.E)$

3) $A.(C+B+E+D)$

4) $D.(F+E.(A+B))$

5) $E.F.(\overline{T.\overline{R}}+D)$

6) $A.(B+C+D)+\overline{A.(\overline{B}+E.(B+C))}$

For questions 7-12, factorise the expressions.

Do **not** simplify the resulting expressions.

There are two correct answers for q12 – make sure you write down both of them.

7) $C.D+C.B$

8) $C.D.B+B.A.E$

9) $A.C+C.B+E.C$

10) $D.F+D.(A+B)$

11) $E.F+E.\overline{F}$

12) $A.1+A.1$

For questions 13-24, simplify the expressions using just the basic rules 1-11 and, if helpful, bracket expansion, factorisation, associative laws and commutative laws. **Show your working out.**

There are many ways of simplifying question 20 that end up getting to the correct answer – make sure you write down at least two of them.

13) $\overline{B}.(D+B)$

14) $C.D.(D.B+C)$

15) $A.(C+B+\overline{A})$

16) $D.(F+D.\left(A+\overline{F}\right))$

17) $E.0.(\overline{T.\overline{R}}+D)$

18) $D.E+E.\overline{D}$

19) $A.B.C+A.B.C$

20) $A.(B+0+D).(A.\left(\overline{B}+D.\left(B+\overline{A}\right)\right))$

21) $\overline{H}+\overline{H}.J$

22) $\overline{\overline{A.\overline{B.C}}+0+A.1.\overline{B}}+1$

23) $S.\overline{\overline{S}}$

24) $A.(A + A).(B + \overline{A})$

25) Prove the Boolean algebra rule $A + A.B = A$. You must use factorisation in your proof.

Simplifying Boolean expressions part 4: De Morgan's Laws

De Morgan's Laws are probably the two most important rules to learn as they can be applied in many expressions which require simplification.

De Morgan's Law 1

$\overline{\overline{A}.\overline{B}} = A + B$

Proof using a truth table:

A	B	$A + B$	$\overline{A}$	$\overline{B}$	$\overline{A}.\overline{B}$	$\overline{\overline{A}.\overline{B}}$
0	0	0	1	1	1	0
0	1	1	1	0	0	1
1	0	1	0	1	0	1
1	1	1	0	0	0	1

As you can see the two highlighted columns are identical to each other, proving that the two expressions are equivalent to each other.

De Morgan's Law 2

$\overline{\overline{A} + \overline{B}} = A.B$

Proof using a truth table:

A	B	$A.B$	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$	$\overline{\overline{A} + \overline{B}}$
0	0	0	1	1	1	0
0	1	0	1	0	1	0
1	0	0	0	1	1	0
1	1	1	0	0	0	1

As you can see the two highlighted columns are identical to each other, proving that the two expressions are equivalent to each other.

Exercise 4

Use De Morgan's laws, and any other rules that will help, to simplify the expressions below **showing your working out** when there is more than one stage to the simplification.

1) $\overline{\overline{C} + \overline{D}}$

2) $\overline{\overline{C} + \overline{F}}$

3) $\overline{\overline{A} + \overline{B}} + A$

4) $\overline{\overline{A}.\overline{D}}$

5) $\overline{\overline{R.D}.\overline{B}}$

6) $\overline{\overline{A}.\overline{E}} + A.D$

7) $\overline{\overline{D} + \overline{B}}.\overline{B}$

8) $\overline{\overline{D}.\overline{E}}.\overline{\overline{D} + \overline{E}}$

For each of the equations below state if they are true (both sides of the equation are equal to each other) or false. If it is false, show what the left-hand side of the equation simplifies to.

9) $\overline{\overline{C} + \overline{D}} = C + D$

10) $\overline{\overline{\overline{F}}} = \overline{F}$

11) $A + A = 1$

12) $V.V = V$

13) $B.\overline{B} = 1$

14) $\overline{\overline{A.D}} = A.D$

15) $B + B.C = C$

16) $\overline{\overline{D}.\overline{E}} + 0 = 0$

17) $\overline{A}.0 = A$

18) $X.(X + \overline{Y}) = X$

19) $\overline{\overline{M}.\overline{N}} = M + N$

20) $0.A + B = 0$

Where can De Morgan's Laws be applied?

They can be applied to any Boolean expression or part of a Boolean expression that contains either an AND or OR logic gate.

Example

In the expression $\overline{A.B} + A.\overline{C}$ there are three places where De Morgan's can be applied because there are three AND/OR logic gates in the expression.

Steps involved in applying De Morgan's Laws

To apply De Morgan's Law to a Boolean expression (or part of a Boolean expression) you should:

1. Switch the logic gate (i.e. if the law is being applied to an AND gate then change the AND gate to an OR; if the law is being applied to an OR gate then change the OR gate to an AND).
2. Add a NOT gate to the expression on the left-hand side of the operator switched in step 1.
3. Add a NOT gate to the expression on the right-hand side of the operator switched in step 1.
4. Add a NOT gate to the whole expression (or part of expression) that the law was being applied to.

Simple example

To apply De Morgan's Law to the expression $A.C$ you:

1. Change the AND logic gate to an OR. $A + C$
2. Apply a NOT to the LHS. $\overline{A} + C$
3. Apply a NOT to the RHS. $\overline{A} + \overline{C}$
4. Apply a NOT to the whole expression. $\overline{\overline{A} + \overline{C}}$

Therefore we can say that $A.C = \overline{\overline{A} + \overline{C}}$

Example

In the expression $\overline{A.B} + A.\overline{C}$ there are three places where De Morgan's can be applied because there are three AND/OR logic gates in the expression.

The steps to apply De Morgan's Law to the **whole expression** are:

1. Change the OR gate to an AND. $\overline{A.B}.A.\overline{C}$
2. Apply a NOT to the expression on the LHS of the changed gate. $\overline{\overline{A.B}}.A.\overline{C}$
3. Apply a NOT to the expression on the RHS of the changed gate. $\overline{\overline{A.B}}.\overline{A.\overline{C}}$
4. Apply a NOT to the whole expression. $\overline{\overline{\overline{A.B}}.\overline{A.\overline{C}}}$

Therefore we can say that $\overline{A.B} + A.\overline{C} = \overline{\overline{\overline{A.B}}.\overline{A.\overline{C}}}$

Note: it is not correct to say $\overline{A.B} + A.\overline{C} = \overline{A.B}.A.\overline{C} = \overline{\overline{A.B}}.A.\overline{C} = \overline{\overline{A.B}}.\overline{A.\overline{C}} = \overline{\overline{\overline{A.B}}.\overline{A.\overline{C}}}$ - the steps involved in De Morgan's Law are not stages of working out and so not all the expressions shown here are equal to each other (e.g. $\overline{\overline{A.B}}.\overline{A.\overline{C}}$ is actually the complete opposite of the final expression).

The steps to apply De Morgan's Law to the part of the expression containing the **first AND** gate are:

1. Change the AND gate to an OR. $\overline{A + B} + A.\overline{C}$
2. Apply a NOT to the expression on the LHS of the changed gate. $\overline{\overline{A} + B} + A.\overline{C}$
3. Apply a NOT to the expression on the RHS of the changed gate. $\overline{\overline{A} + \overline{B}} + A.\overline{C}$
4. Apply a NOT to the part of the expression the law is being applied to. $\overline{\overline{\overline{A} + \overline{B}}} + A.\overline{C}$

Therefore we can say that $\overline{A.B} + A.\overline{C} = \overline{\overline{\overline{A} + \overline{B}}} + A.\overline{C}$

The steps to apply De Morgan's Law to the part of the expression containing the **second AND** gate are:

1. Change the AND gate to an OR. $\overline{A.B} + A + \overline{C}$
2. Apply a NOT to the expression on the LHS of the changed gate. $\overline{A.B} + \overline{A} + \overline{C}$
3. Apply a NOT to the expression on the RHS of the changed gate. $\overline{A.B} + \overline{A} + \overline{\overline{C}}$
4. Apply a NOT to the part of the expression the law is being applied to. $\overline{A.B} + \overline{\overline{A} + \overline{\overline{C}}}$

Therefore we can say that $\overline{A.B} + A.\overline{C} = \overline{A.B} + \overline{\overline{A} + \overline{\overline{C}}}$

So using De Morgan's Laws we have found three expressions that are equivalent to our original expression $\overline{A.B} + A.\overline{C}$:

$$\overline{\overline{\overline{A.B}}.\overline{A.\overline{C}}}$$

$$\overline{\overline{\overline{A} + \overline{B}}} + A.\overline{C}$$

$$\overline{A.B} + \overline{\overline{A} + \overline{\overline{C}}}$$

Note: all three of these equivalent expressions could be simplified by cancelling some NOTs. Also, in these examples applying De Morgan's Law was not very helpful as the resulting expressions are all more complex than the original expression! This shows that just because you can apply De Morgan's Laws does not mean that you always should.

Warning

One of the most common mistakes when applying De Morgan's Laws is to forget about the order of priority of the Boolean operators.

In the Boolean expression $A.B + C$ there are two places where De Morgan's Laws can be used. The two correct applications of the Laws are:

$$A.B + C = \overline{\overline{A.B}.\overline{C}}$$

$$A.B + C = \overline{\overline{A} + \overline{B}} + C$$

A common mistake would be to take the LHS of the AND to be *A* and the RHS of the AND to be $B + C$, but AND takes priority over OR so the following is **wrong**:

$$A.B + C = \overline{\overline{A} + \overline{B + C}}$$

Exercise 5

For questions 1–8, state how many different places there are where De Morgan's Laws can be applied in the given expression.

1) $C + B$

2) $\overline{\overline{C} + \overline{F}}$

3) $\overline{\overline{A} + \overline{B}} + A$

4) $\overline{\overline{A}.\overline{D}}$

5) $\overline{\overline{R.D}.\overline{B}}$

6) $\overline{\overline{A}.\overline{E}} + A.D$

7) $\overline{\overline{D} + \overline{B}}.\overline{B}$

8) $\overline{\overline{D}.\overline{E}}.\overline{\overline{D} + \overline{E}}$

For each of the expressions in questions 9-14, show the results of applying De Morgan's Laws in each of the places they can be applied. Do not use any other simplification rules.

9) $C + D + E$

10) $\overline{F.E}$

11) $A.B + \overline{C}$

12) $(V + \overline{U + W}).X$

13) $\overline{\overline{B}.\overline{C}}$

14) $A.\overline{D}$

Substitution – a technique that can help when using De Morgan’s Laws

Sometimes it can be hard to work out how to apply De Morgan’s Laws to part of a Boolean expression, substitution is a technique that can help. When using the substitution technique the identifier for the new variable you use in the expression must not be the same as an identifier already used elsewhere in the expression.

Example

In the expression $\overline{\overline{A.B} + C}$ we could replace the $\overline{A.B}$ with a variable e.g. *D* (note: could not use *A*, *B* or *C* as these are already used in the original expression). This would give the expression $\overline{D + C}$. We can then apply De Morgan’s Law to this expression:

$$\overline{D + C} = \overline{\overline{\overline{D}.\overline{C}}}$$

Having applied De Morgan’s Law to the expression we can now substitute $\overline{A.B}$ back into the changed expression in place of the *D*.

$$\overline{D + C} = \overline{\overline{\overline{D}.\overline{C}}} = \overline{\overline{\overline{\overline{A.B}}.\overline{C}}}$$

The expression can then be simplified.

$$\overline{D + C} = \overline{\overline{\overline{D}.\overline{C}}} = \overline{\overline{\overline{\overline{A.B}}.\overline{C}}} = A.B.\overline{C}$$

You never need to use the substitution technique (if you are able to, you can always apply De Morgan’s Law to the relevant part of the expression without substituting) but it can make expressions easier to simplify.

Exercise 6

For questions 1-34, simplify each Boolean expression using the basic rules, De Morgan's Laws and any other relevant techniques as needed.

1) $(\overline{A}+B).(A+\overline{B})$

2) $A.B+B$

3) $A.B+A.\overline{B}$

4) $B.(A+\overline{A})$

5) $\overline{\overline{A}+\overline{B.A}}$

6) $\overline{1.B}$

7) $(\overline{A.B})+(\overline{A.\overline{B}})$

8) $\overline{A+B+\overline{A}}$

9) $A.(B+1)$

10) $(X+Y).(X+\overline{Y})$

11) $\overline{\overline{A}.\overline{B}}+A+\overline{B}$

12) $(\overline{\overline{A}+\overline{C}}).(\overline{A+B})$

13) $\overline{(\overline{A+B}).(\overline{\overline{A}+C})}$

14) $\overline{D.\overline{E}+\overline{E}.(D+\overline{D})}$

15) $\overline{D.(E+\overline{D})}$

16) $\overline{\overline{A}.B+A.\overline{(A+\overline{B})}}$

17) $B.(A+\overline{B})$

18) $\overline{\overline{D+\overline{E}}+\overline{E}}$

19) $(\overline{A}+A.B).\overline{B}$

20) $\overline{\overline{A}+\overline{B}}+B.\overline{A}$

21) $\overline{\overline{D}+\overline{E}}$

22) $\overline{X.\overline{Y}+\overline{X}.\overline{Y}+\overline{X}.Y}$

23) $\overline{B}.\overline{\overline{A}+\overline{B}}$

24) $\overline{\overline{D}+\overline{\overline{E}}}$

25) $X.(\overline{X}+Y)$

26) $\overline{\overline{A}.\overline{B}}$

27) $\overline{(D.\overline{E}).D}+\overline{E}$

28) $A.B.\overline{C}+A.\overline{C}$

29) $\overline{\overline{A} + \overline{B}} + A.\overline{B}$

30) $\overline{A} + \overline{B + A}$

31) $X + \overline{\overline{Y}.\overline{Z}}$

32) $\overline{\overline{X}.\overline{Y}} + \overline{X}$

33) $\overline{X.\overline{Y} + \overline{X}.\overline{Y}}$

34) $\overline{A.C + A.B + B.\overline{C} + \overline{A}.C}$

35) Prove, using the rules of Boolean algebra, that: $1 \oplus A = \overline{A}$.

36) Prove, using the rules of Boolean algebra, that: $0 \oplus A = A$.

Answers

Exercise 1 answers

1) $A + A = A$

(using basic rule 3)

2) $B + B = B$

(using basic rule 3)

3) $A.B + A.B = A.B$

(using basic rule 3 – an expression ORed with itself results in just that expression)

4) $D.F + D.F + G = D.F + G$

(using basic rule 3, it is then not possible to simplify $D.F + G$ any further)

5) $D.E + A.\overline{B} + D.E$

$= D.E + D.E + A.\overline{B}$

(using the commutative laws – showing this stage of the working out is not really needed)

$= D.E + A.\overline{B}$

(using basic rule 3)

6) $A.A = A$

(using basic rule 7)

7) $H.H = H$

(using basic rule 7)

8) $(A + B).(A + B) = A + B$

(using basic rule 7)

9) $X.Y.X$

$= X.X.Y$

(using the commutative laws – showing this stage of the working out is not really needed)

$= X.Y$

(using basic rule 7)

10) $B.1 = B$

(using basic rule 6)

11) $C + 0 = C$

(using basic rule 1)

12) $A.B.A.C.1$

$= A.B.A.C$

(using basic rule 6)

$= A.A.B.C$

(using the commutative laws – showing this stage of the working out is not really needed)

$= A.B.C$

(using basic rule 7)

13) $D + 0 = D$

(using basic rule 1)

14) $A.B + 0 = A.B$

(using basic rule 1)

15) $E.1 = E$

(using basic rule 6)

16) $E.0 = 0$

(using basic rule 5)

17) $(A + A.B).0 = 0$

(using basic rule 5 – anything ANDed with 0 (False) is 0)

Note: there are other ways of simplifying this expression but they use more than just the basic rules 1-9. They also require more steps in the simplification process and so this is the shortest method to use. As an additional exercise you can try to simplify this expression using basic rule 10 and expanding brackets (as well as basic rules 1-9).

18) $A.B.1 = A.B$

(using basic rule 6)

19) $\overline{\overline{D}} = D$

(using basic rule 9)

20) $\overline{\overline{B + B}}$

$= B + B$

(using basic rule 9)

$= B$

(using basic rule 3)

21) $\overline{C + \overline{\overline{D}}} = \overline{C + D}$

(using basic rule 9)

22) $A + \overline{B.\overline{B}}$

$= A + \overline{0}$

(using basic rule 8)

$= A + 1$

(because applying the NOT operator to false (0) gives true (1))

$= 1$

(using basic rule 2)

23) $\overline{A + \overline{B}}$

This expression is already in its simplest form, none of the simplification rules can be applied so it cannot be simplified further e.g. you can't use basic rule 9 to cancel out the NOTs as they are not over the same part of the expression.

24) $A + A + 1$

There are several ways of simplifying this expression.

The quickest way is to use the associative laws (as all operators in the expression are the same) to realise that this expression can be thought of as being:

$(A + A) + 1$

Which can then be simplified using basic rule 2 to:

1

An alternative method is to use basic rule 3 (or basic rule 2) to simplify $A + A + 1$ to:

$A + 1$

And then simplify that using basic rule 2 to:

1

25) $A.B.1.0$

Again, there are several different ways of simplifying this expression.

The quickest method is to use the associative laws to identify that the expression can be rewritten as:

$(A.B.1).0$

and then use basic rule 5 to simplify that expression to:

0

An alternative method would be to use basic rule 6 to simplify the expression $A.B.1.0$ to:

$A.B.0$

And then use basic rule 5 to simplify that expression to:

$A.0$

And then use basic rule 5 again to simplify that expression to:

0

26) $A + \overline{\overline{\overline{B}}} = A + \overline{B}$

(using basic rule 9)

Note: you can only cancel out **even** numbers of NOTs (if they are over the same part of the expression). You cannot cancel out all of the NOTs if there are an odd number of NOTs over part of an expression.

27)

E	**0**	**E.0**
0	0	0
1	0	0

28)

E	**1**	**E.1**
0	1	0
1	1	1

29)

E	**E**	**E.E**
0	0	0
1	1	1

30)

E	$\overline{E}$	$E.\overline{E}$
0	1	0
1	0	0

31)

E	**E**	**E+E**
0	0	0
1	1	1

32)

E	$\overline{E}$	$\overline{\overline{E}}$
0	1	0
1	0	1

33

E	**1**	**E+1**
0	1	1
1	1	1

34)

E	**0**	**E+0**
0	0	0
1	0	1

35)

E	$\overline{E}$	$E+\overline{E}$
0	1	1
1	0	1

Exercise 2 answers

1) $C + C.D = C$

(using basic rule 10)

2) $D + C.D.B$

$= D + D.(C.B)$

(using commutative and associative laws – showing this stage of the working out is not really needed)

$= D$

(using basic rule 10)

3) $A.(C + A)$

$= A.(A + C)$

(using commutative laws – showing this stage of the working out is not really needed)

$= A$

(using basic rule 11)

4) $D.F + D.1$

$= D.F + D$

(using basic rule 6)

$= D + D.F$

(using commutative laws – showing this stage of the working out is not really needed)

$= D$

(using basic rule 10)

5) $E.F.(E.F + D)$

$= E.F$

(using basic rule 11: $A.(A + B) = A$ - with D being equivalent to B and E.F equivalent to A)

6) $A.A + A.1 + B.\overline{B}$

$= A.A + A.1 + 0$

(using basic rule 8)

$= A.A + A.1$

(using basic rule 1)

$= A.A + A$

(using basic rule 6)

$= A$

(using basic rule 10 and commutative laws)

There are several alternative answers to the one shown. Here is another possible method:

$A.A + A.1 + B.\overline{B}$

$= A + A.1 + B.\overline{B}$

(using basic rule 7)

$= A + A + B.\overline{B}$

(using basic rule 6)

$= A + B.\overline{B}$

(using basic rule 3)

$= A + 0$

(using basic rule 8)

$= A$

(using basic rule 1)

7) $A.B.B.C + A.0.B + A.C.B.D$

$= A.B.B.C + 0 + A.C.B.D$

(using basic rule 5 and commutative laws)

$= A.B.B.C + A.C.B.D$

(using basic rule 1)

$= A.C.B + A.C.B.D$

(using basic rule 7 and commutative laws)

$= A.C.B$

(using basic rule 10)

Note: there are many other ways of answering this question

8) $A.B.\left(0+\overline{A}\right)+1$

$=1$

(using basic rule 2)

Note: there are clearly several other rules that could have been used to simplify this expression (e.g. basic rule 1) but this is by far the quickest method.

9) $\overline{D}.\left(\overline{E}+\overline{D}\right)$

$=\overline{D}.(\overline{D}+\overline{E})$

(using commutative laws)

$\overline{D}$

(using basic rule 10)

10) $F.(F+1)$

$=F$

(using basic rule 11)

Here is another possible method:

$F.(F+1)$

$=F.1$

(using basic rule 2)

$=F$

(using basic rule 6)

11) $E.F+F.G.E$

$=E.F+E.F.G$

(using commutative laws)

$=E.F$

(using basic rule 10)

12) $\overline{\overline{B.\overline{C}}} + \overline{C}$

$= B.\overline{C} + \overline{C}$

(using basic rule 9)

$= \overline{C}$

(using basic rule 10)

13) $A.B.(A + A.B)$

$= A.B.A$

(using basic rule 10)

$= A.B$

(using basic rule 7)

Here is another possible method:

$A.B.(A + A.B)$

$= A.B.(A.B + A)$

(using commutative laws – showing this stage of the working out is not really needed)

$= A.B$

(using basic rule 11)

14) $A.B.\overline{C} + A.\overline{C}$

$= A.\overline{C} + A.\overline{C}.B$

(using commutative laws – showing this stage of the working out is not really needed)

$= A.\overline{C}$

(using basic rule 10)

15) $A.\overline{D}.(C + A.\overline{D})$

$= A.\overline{D}$

(using basic rule 11 and commutative laws)

16) $\overline{C.\overline{D}.\overline{D}.E} + \overline{C.E}$

$= \overline{C.\overline{D}.E} + \overline{C.E}$

(using basic rule 7)

Note: this expression can be simplified further by using De Morgan's Laws to $\overline{C.\overline{D}.E}$ (see later) but cannot be simplified further using just the basic rules 1-11.

17) $A + B.A + B.A.C$

$= A$

(using two applications of basic rule 10)

18) $E.(E + 1)$

$= E.1$

(using basic rule 2)

$= E$

(using basic rule 6)

Here is another possible method:

$E.(E + 1) = E$

(using basic rule 11)

19) $\overline{D} + D.0$

$= \overline{D} + 0$

(using basic rule 5)

$= \overline{D}$

(using basic rule 1)

20) $A + A.\overline{A} = A$

(using basic rule 10)

Here is another possible method:

$A + A.\overline{A}$

$= A + 0$

(using basic rule 8)

$= A$

(using basic rule 1)

21) $\overline{\overline{\overline{B} + \overline{B}.\overline{B}}}$

$= \overline{B} + \overline{B}.\overline{B}$

(using basic rule 9)

$= \overline{B}$

(using basic rule 10)

22) $C.A.B.(A.B + D)$

$= C.(A.B.(A.B + D))$

(using associative laws – showing this stage of the working out is not really needed)

$= C.A.B$

(using basic rule 11 and then removing unnecessary set of brackets)

23) $(A.\overline{G} + A.1).A$

$= (A.\overline{G} + A).A$

(using basic rule 6)

$= A$

(using basic rule 11)

24) $B.A + A = A$

(using basic rule 10)

25) $A.(A+B).\overline{1}$

$= A.0$

(using basic rule 11 and applying NOT to 1)

$= 0$

(using basic rule 5)

26) $A.B.(A.B+E)+\overline{\overline{\overline{A.\overline{\overline{B}}}}}$

$= A.B.(A.B+E)+\overline{A.B}$

(using basic rule 9 twice)

$= A.B+\overline{A.B}$

(using basic rule 11)

$= 1$

(using basic rule 4)

27)

B	$\overline{\boldsymbol{B}}$	$\boldsymbol{B.\overline{B}}$
0	1	0
1	0	0

28)

B	**1**	**B.1**
0	1	0
1	1	1

29)

B	$\overline{\boldsymbol{B}}$	$\overline{\overline{\boldsymbol{B}}}$
0	1	0
1	0	1

30)

$\boldsymbol{B}$	$\overline{\boldsymbol{B}}$	$\boldsymbol{B+\overline{B}}$
0	1	1
1	0	1

31)

B	B	B+B
0	0	0
1	1	1

32)

B	1	B+1
0	1	1
1	1	1

33)

B	B	B.B
0	0	0
1	1	1

34)

B	0	B+0
0	0	0
1	0	1

35)

B	0	B.0
0	0	0
1	0	0

Exercise 3 answers

1) $C.D + C.B$

2) $C.D.B + C.D.A.E$

3) $A.C + A.B + A.E + A.D$

4) $= D.(F + E.A + E.B)$

$= D.F + D.E.A + D.E.B$

5) $E.F.\overline{T.\overline{R}} + E.F.D$

6) $= A.(B + C + D) + \overline{A.(\overline{B} + E.B + E.C)}$

$= A.(B + C + D) + \overline{A.\overline{B} + A.E.B + A.E.C}$

$= A.B + A.C + A.D + \overline{A.\overline{B} + A.E.B + A.E.C}$

7) $C.(D + B)$

8) $B.(C.D + A.E)$

9) $C.(A + B + E)$

10) $D.(F + A + B)$

11) $E.(F + \overline{F})$

12) $A.(1 + 1)$

or

$= 1.(A + A)$

13) $\overline{B}.(D + B)$

$= \overline{B}.D + \overline{B}.B$

(expanding brackets)

$= \overline{B}.D + 0$

(using basic rule 8)

$= \overline{B}.D$

(using basic rule 1)

14) $C.D.(D.B + C)$

$= C.D.D.B + C.D.C$

(expanding brackets)

$= C.D.B + C.D$

(using basic rule 7 twice)

$= C.D$

(using basic rule 10)

15) $A.(C + B + \overline{A})$

$= A.C + A.B + A.\overline{A}$

(expanding brackets)

$= A.C + A.B$

(using basic rule 8 and then basic rule 1)

$= A.(C + B)$

(factorising)

16) $D.(F + D.(A + \overline{F}))$

$= D.(F + D.A + D.\overline{F})$

(expanding brackets)

$= D.F + D.D.A + D.D.\overline{F}$

(expanding brackets)

$= D.F + D.\overline{F} + D.A$

(commutative laws and two applications of basic rule 7)

$= D.(F + \overline{F}) + D.A$

(factorising)

$= D.1 + D.A$

(basic rule 4)

$= D + D.A$

(basic rule 6)

$= D$

(basic rule 10)

17) $E.0.(\overline{T.\overline{R}} + D)$

$= 0$

(using two applications of basic rule 5)

18) $D.E + E.\overline{D}$

$= E.(D + \overline{D})$

(factorising)

$= E.1$

(basic rule 4)

$= E$

(basic rule 6)

19) $A.B.C + A.B.C = A.B.C$

(using basic rule 3)

20) $A.(B + 0 + D).(A.\left(\overline{B} + D.\left(B + \overline{A}\right)\right))$

$= A.(B + D).(A.\left(\overline{B} + D.B + D.\overline{A}\right))$

(using basic rule 1 and expanding brackets)

$= (A.B + A.D).(A.\overline{B} + A.D.B + A.\overline{A}.D)$

(expanding brackets twice)

$= (A.B + A.D).(A.\overline{B} + A.D.B)$

(using basic rule 8 and then basic rule 5)

$= A.B.A.\overline{B} + A.D.A.\overline{B} + A.B.A.D.B + A.D.A.D.B$

(expanding brackets)

$= A.B.\overline{B} + A.D.\overline{B} + A.D.B + A.D.B$

(multiple applications of basic rule 7)

$= A.B.\overline{B} + A.D.\overline{B} + A.D.B$

(using basic rule 3)

$= A.D.\overline{B} + A.D.B$

(using basic rule 8 and then basic rule 5)

$= A.D.(\overline{B} + B)$

(factorising)

$= A.D$

(using basic rule 4 and then basic rule 6)

21) $\overline{H} + \overline{H}.J = \overline{H}$

(using basic rule 10)

22) $\overline{\overline{A.\overline{B.C} + 0} + A.1.\overline{B}} + 1 = 1$

(using basic rule 2)

23) $S.\overline{\overline{S}}$

$= S.S$

(using basic rule 9)

$= S$

(using basic rule 7)

24) $A.(A + A).(B + \overline{A})$

$= A.A.(B + \overline{A})$

(using basic rule 3)

$= A.(B + \overline{A})$

(using basic rule 7)

$= A.B + A.\overline{A}$

(expanding brackets)

$= A.B$

(using basic rule 8 and then basic rule 1)

25) $A + A.B = A.1 + A.B = A.(1 + B) = A.1 = A$

Exercise 4 answers

From now on the answers to the exercises will just show the answers (and working out); they will not include references to which rules/laws have been used in the simplification process.

1) $\overline{\overline{C} + \overline{D}} = C.D$

2) $\overline{\overline{C} + \overline{F}} = C.F$

3) $\overline{\overline{A} + \overline{B}} + A = A.B + A = A$

4) $\overline{\overline{A}.\overline{D}} = A + D$

5) $\overline{\overline{R.D}.\overline{B}} = \overline{\overline{\overline{\overline{R.D}} + \overline{\overline{B}}}} = R.D + B$

6) $\overline{\overline{A}.\overline{E}} + A.D = A + E + A.D = A + A.D + E = A + E$

7) $\overline{\overline{D} + \overline{B}}.\overline{B} = D.B.\overline{B} = D.0 = 0$

8) $\overline{\overline{D}.\overline{E}}.\overline{\overline{D} + \overline{E}} = (D + E).D.E = D.D.E + E.D.E = D.E + E.D = D.E$

9) $\overline{\overline{C} + \overline{D}} = C + D$

This equation is false. $\overline{\overline{C} + \overline{D}}$ simplifies to $C.D$.

10) $\overline{\overline{\overline{F}}} = \overline{F}$

This equation is true.

11) $A + A = 1$

This equation is false. $A + A$ simplifies to A.

12) $A + A = 1$

This equation is true.

13) $B.\overline{B} = 1$

This equation is false. $B.\overline{B}$ simplifies to 0.

14) $\overline{\overline{A.D}} = A.D$

This equation is true.

15) $B + B.C = C$

This equation is false. $B + B.C$ simplifies to B.

16) $\overline{\overline{D}.\overline{E}} + 0 = 0$

This equation is false. $\overline{\overline{D}.\overline{E}} + 0$ simplifies to $\overline{\overline{D}.\overline{E}}$ which can then be further simplified to $D + E$.

17) $\overline{A}.0 = A$

This equation is false. $\overline{A}.0$ simplifies to 0.

18) $X.\left(X + \overline{Y}\right) = X$

This equation is true.

19) $\overline{\overline{M}.\overline{N}} = M + N$

This equation is true.

20) $0.A + B = 0$

This equation is false. $0.A + B$ simplifies to $0 + B$ which further simplifies to B.

Exercise 5 answers

1) One

2) One

3) Two

4) One

5) Two

6) Three

7) Two

8) Three

9) $\overline{\overline{C}.\overline{D}} + E$

$C + \overline{\overline{D}.\overline{E}}$

$\overline{\overline{C}.\overline{D + E}}$

$\overline{\overline{C + D}.\overline{E}}$

Note: $\overline{\overline{C}.\overline{E}} + D$ *and* $\overline{\overline{C + E}.\overline{D}}$ *, could also be considered correct answers (due to commutative laws)*

10) $\overline{\overline{\overline{F} + \overline{E}}}$

11) $\overline{\overline{A} + \overline{B}} + \overline{C}$

$$\overline{\overline{A.B}.\overline{\overline{C}}}$$

12) $$\overline{\overline{V+\overline{U+W}}+\overline{X}}$$

$$\overline{\overline{V}.\overline{\overline{U+W}}}.X$$

$$\left(V+\overline{\overline{\overline{U}.\overline{W}}}\right).X$$

13) $$\overline{\overline{\overline{\overline{B}}+\overline{\overline{C}}}}$$

14) $$\overline{\overline{A}+D}$$

Exercise 6 answers

In these answers some alternative methods are shown, when appropriate. For some of the more complex questions there could be other, correct, alternative methods which have not been shown.

1) $(\overline{A}+B).(A+\overline{B}) = \overline{A}.A + \overline{A}.\overline{B} + B.A + B.\overline{B} = 0 + \overline{A}.\overline{B} + B.A + 0 = \overline{A}.\overline{B} + B.A$

Note: this final expression could also be written as $\overline{A \oplus B}$.

2) *B*

3) $A.B + A.\overline{B} = A.(B+\overline{B}) = A.1 = A$

4) $B.(A+\overline{A}) = B.1 = B$

5) $\overline{\overline{A}+\overline{B.A}} = \overline{\overline{A}+\overline{\overline{\overline{B}+\overline{A}}}} = \overline{\overline{A}+\overline{A}+\overline{B}} = \overline{\overline{A}+\overline{B}} = \overline{\overline{\overline{\overline{A}}.\overline{\overline{B}}}} = A.B$

Alternative method:

$\overline{\overline{A}+\overline{B.A}} = \overline{\overline{\overline{\overline{A}}.\overline{\overline{B.A}}}} = A.A.B = A.B$

6) $\overline{B}$

7) $(\overline{A.B}) + (\overline{A.\overline{B}}) = \overline{\overline{\overline{A}+\overline{B}}} + \overline{A.\overline{B}} = \overline{A}+\overline{B}+\overline{A.\overline{B}} = \overline{A}+\overline{B}+\overline{\overline{\overline{A}+\overline{\overline{B}}}}$

$= \overline{A}+\overline{B}+\overline{A}+B = \overline{A}+\overline{B}+B = \overline{A}+1 = 1$

Alternative method:

$(\overline{A.B}) + (\overline{A.\overline{B}}) = \overline{\overline{\overline{A.B}}.\overline{\overline{A.\overline{B}}}} = \overline{A.B.A.\overline{B}} = \overline{A.B.\overline{B}} = \overline{A.0} = \overline{0} = 1$

8) $\overline{A+B+\overline{A}} = \overline{1+B} = \overline{1} = 0$

9) $A.(B+1)=A.1=A$

10) $(X+Y).(X+\overline{Y})=X.X+X.\overline{Y}+Y.X+Y.\overline{Y}=X+X.\overline{Y}+Y.X+0$
$=X+X.\overline{Y}+Y.X=X+X.Y=X$

Alternative method:

$(X+Y).(X+\overline{Y})=X.X+X.\overline{Y}+Y.X+Y.\overline{Y}=X.(X+\overline{Y}+Y)+Y.\overline{Y}=X+0=X$

11) $\overline{\overline{A}.\overline{B}}+A+\overline{B}=A+B+A+\overline{B}=A+A+1=1$

12) $\left(\overline{\overline{A}+\overline{C}}\right).\left(\overline{A+B}\right)=A.C.\overline{A+B}=A.C.\overline{\overline{\overline{A}.\overline{B}}}=A.\overline{A}.C.\overline{B}=0.C.\overline{B}=0$

Alternative method:

$\left(\overline{\overline{A}+\overline{C}}\right).\left(\overline{A+B}\right)=\overline{\left(\overline{\overline{\overline{A}+\overline{C}}}\right)+\left(\overline{\overline{A+B}}\right)}=\overline{\overline{A}+\overline{C}+A+B}=\overline{1+C+B}=\overline{1}=0$

13) $\overline{\left(\overline{A+B}\right).\left(\overline{\overline{A}+C}\right)}=\overline{\overline{\overline{\overline{A+B}}+\overline{\overline{\overline{A}+C}}}}=A+B+\overline{A}+C=1+B+C=1$

Alternative answer:

$\overline{\left(\overline{A+B}\right).\left(\overline{\overline{A}+C}\right)}=\overline{\overline{\overline{\overline{A}.\overline{B}}}.\overline{\overline{A}+C}}=\overline{\overline{A}.\overline{B}.\overline{\overline{A}+C}}=\overline{\overline{A}.\overline{B}.\overline{\overline{\overline{\overline{A}}.\overline{C}}}}=\overline{\overline{A}.\overline{B}.A.\overline{C}}=\overline{0.\overline{B}.\overline{C}}=\overline{0}$
$=1$

14) $\overline{D.\overline{E}+\overline{E}.(D+\overline{D})}=\overline{D.\overline{E}+\overline{E}.1}=\overline{\overline{E}.(D+1)}=\overline{\overline{E}.1}=\overline{\overline{E}}=E$

15) $\overline{D.(E+\overline{D})}=\overline{D.E+D.\overline{D}}=\overline{D.E+0}=\overline{D.E}$

Note: this answer is D NAND E

16) $\overline{\overline{A}.B + A.\overline{(A+\overline{B})}} = \overline{\overline{A}.B + A.(\overline{\overline{\overline{A}.\overline{\overline{B}}}})} = \overline{\overline{A}.B + A.\overline{A}.B} = \overline{\overline{A}.B + 0.B} = \overline{\overline{A}.B} = \overline{\overline{\overline{\overline{A}} + \overline{B}}}$
$= A + \overline{B}$

Alternative answer:

$\overline{\overline{A}.B + A.\overline{(A+\overline{B})}} = \overline{\overline{A}.B + \overline{\overline{A} + \overline{\overline{(A+\overline{B})}}}} = \overline{\overline{A}.B + \overline{\overline{A} + A + \overline{B}}} = \overline{\overline{A}.B + \overline{1 + \overline{B}}}$
$= \overline{\overline{A}.B + \overline{1}} = \overline{\overline{A}.B + 0} = \overline{\overline{A}.B} = \overline{\overline{\overline{\overline{A}} + \overline{B}}} = A + \overline{B}$

17) $B.\left(A+\overline{B}\right) = B.A + B.\overline{B} = B.A + 0 = B.A$

18) $\overline{\overline{D+\overline{E}}+\overline{E}} = \overline{\overline{\overline{\overline{D+\overline{E}}}.\overline{\overline{E}}}} = \left(D+\overline{E}\right).E = D.E + \overline{E}.E = D.E + 0 = D.E$

19) $\left(\overline{A} + A.B\right).\overline{B} = \overline{A}.\overline{B} + A.B.\overline{B} = \overline{A}.\overline{B} + A.0 = \overline{A}.\overline{B} + 0 = \overline{A}.\overline{B} = \overline{A+B}$

20) $\overline{\overline{A}+\overline{B}} + B.\overline{A} = A.B + B.\overline{A} = B.\left(A+\overline{A}\right) = B.1 = B$

21) $D.E$

22) $\overline{X.\overline{Y} + \overline{X}.\overline{Y} + \overline{X}.Y} = \overline{\overline{Y}.\left(X+\overline{X}\right) + \overline{X}.Y} = \overline{\overline{Y}.1 + \overline{X}.Y} = \overline{\overline{Y} + \overline{X}.Y} = \overline{\overline{Y} + \overline{(X+\overline{Y})}}$
$= Y.\left(X+\overline{Y}\right) = Y.\overline{Y} + Y.X = 0 + Y.X = Y.X$

23) $\overline{B}.\overline{\overline{A}+\overline{B}} = \overline{B}.A.B = 0.A = 0$

Alternative answer:

$\overline{B}.\overline{\overline{A}+\overline{B}} = \overline{\overline{\overline{B}} + \overline{\overline{\overline{A}+\overline{B}}}} = \overline{B + \overline{A} + \overline{B}} = \overline{1+\overline{A}} = \overline{1} = 0$

24) $\overline{\bar{D} + \bar{\bar{E}}} = \overline{\bar{D} + E} = \overline{\overline{\bar{\bar{D}}.\bar{E}}} = D.\bar{E}$

25) $X.\left(\bar{X} + Y\right) = X.\bar{X} + X.Y = 0 + X.Y = X.Y$

26) $A + B$

27) $\overline{(D.\bar{E})}.D + \bar{E} = \overline{D.\bar{E}} + \bar{E} = \bar{D} + E + \bar{E} = \bar{D} + 1 = 1$

28) $A.B.\bar{C} + A.\bar{C} = A.\bar{C} + A.\bar{C}.B = A.\bar{C}$

Alternative answer:

$A.B.\bar{C} + A.\bar{C} = A.\bar{C}.B + A.\bar{C}.1 = A.\bar{C}.(B + 1) = A.\bar{C}.1 = A.\bar{C}$

29) $\overline{\bar{A} + \bar{B}} + A.\bar{B} = A.B + A.\bar{B} = A.\left(B + \bar{B}\right) = A.1 = A$

30) $\bar{A} + \overline{B + A} = \bar{A} + \bar{A}.\bar{B} = \bar{A}$

31) $X + Y + Z$

32) $\overline{\bar{X}.\bar{Y}} + \bar{X} = X + Y + \bar{X} = 1 + Y = 1$

33) $\overline{X.\bar{Y} + \bar{X}.\bar{Y}} = \overline{\bar{Y}.(X + \bar{X})} = \overline{\bar{Y}.1} = \bar{\bar{Y}} = Y$

Alternative answer:

$$\begin{aligned}\overline{X.\bar{Y} + \bar{X}.\bar{Y}} &= \overline{\overline{\overline{X.\bar{Y}}.\overline{\bar{X}.\bar{Y}}}} = \overline{X.\bar{Y}}.\overline{\bar{X}.\bar{Y}} = \left(\bar{X} + Y\right).\overline{\bar{X}.\bar{Y}} = \left(\bar{X} + Y\right).(X + Y) \\ &= \bar{X}.X + \bar{X}.Y + Y.X + Y.Y = 0 + \bar{X}.Y + Y.X + Y \\ &= Y + Y.\bar{X} + Y.X = Y + Y.X = Y\end{aligned}$$

Alternative answer:

$$\overline{X.\overline{Y} + \overline{X}.\overline{Y}} = \overline{\overline{\overline{X} + \overline{\overline{Y}}} + \overline{X}.\overline{Y}} = \overline{\overline{\overline{X} + \overline{\overline{Y}}} + \overline{\overline{\overline{X}} + \overline{\overline{Y}}}} = \overline{\overline{\overline{X} + Y} + \overline{X + Y}} = \overline{\overline{\overline{\overline{\overline{X} + Y}}}.\overline{\overline{X + Y}}}$$
$$= \left(\overline{X} + Y\right).(X + Y) = \overline{X}.X + \overline{X}.Y + Y.X + Y.Y$$
$$= 0 + \overline{X}.Y + Y.X + Y = Y + Y.\overline{X} + Y.X = Y + Y.X = Y$$

34) $\overline{A.C + A.B + B.\overline{C} + \overline{A}.C} = \overline{C.\left(A + \overline{A}\right) + A.B + B.\overline{C}} = \overline{C.1 + A.B + B.\overline{C}}$

$$= \overline{C + A.B + B.\overline{C}} = \overline{\overline{C}.B + C + A.B} = \overline{\overline{C + \overline{B}} + C + A.B}$$
$$= \overline{\overline{\left(C + \overline{B}\right).\overline{C}} + A.B} = \overline{\overline{C.\overline{C} + \overline{B}.\overline{C}} + A.B} = \overline{\overline{0 + \overline{B}.\overline{C}} + A.B}$$
$$= \overline{\overline{\overline{B}.\overline{C}} + A.B} = \overline{C + B + B.A} = \overline{C + B}$$

Note: this answer is C NOR B

35) $1 \oplus A = A.\overline{1} + \overline{A}.1 = A.0 + \overline{A}.1 = 0 + \overline{A} = \overline{A}$

36) $0 \oplus A = A.\overline{0} + \overline{A}.0 = A.1 + \overline{A}.0 = A + 0 = A$